ISSN 1995-4239, Numerical Analysis and Applications, 2008, Vol. 1, No. 1, pp. 1–16. © Pleiades Publishing, Ltd., 2008.
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Two Methods for Analyzing Stochastic Multistructural Systems with Distributed Structure Changes

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Received September 26, 2006; in final form, January 19, 2007

Abstract—Stochastic multistructural systems with distributed structure changes are analyzed based on two methods: the statistical simulation method and the spectral method. Algorithms for solving the analysis problem are outlined. Numerical examples are furnished to illustrate efficiency of the methods.

DOI: 10.1134/S1995423908010011

Key words: *stochastic multistructural systems, switching diffusions, analysis problem, generalized Fokker–Planck–Kolmogorov equations, statistical simulation method, spectral method.*

1. INTRODUCTION

Modern problems of controlling technical objects are described by mathematical models given by different equations on random time intervals, that is, use is made of models for stochastic multistructural systems. Such are also called *random structure systems*. The models make it possible to take into account random factors, various operating modes, abrupt external effects, or possible malfunctions of model elements.

Random structure systems are exemplified by the following: aircraft approach control systems; signal searching, trapping, and tracking systems in navigation and flight control over air vehicles; systems of combined target guiding; control systems with possible failures and faults [1, 2]. There are a number of factors that cause system structure changes: for instance, a subsystem failure, information supply breaks in the control loop, adaptation to environmental conditions, abrupt noise variation resulting from natural or artificial external effects, etc. Thus, random structure systems are mathematical models for multimode automatic control stochastic systems. Typically, such systems are apt to abruptly change their individual parameters or the entire structure (i.e., the totality of functional elements and relations between them) at random time instants.

An analytical solution for such systems can be found seldom if ever. Therefore, approximate methods have received wide use. These methods can be divided into two groups. One group involves methods that are based on direct simulation of a control system under random perturbations with subsequent statistical processing of results. In methods of the other group, an initial stochastic problem is changed into a deterministic one—for instance, in solving equations for the probability density or moment characteristics of the processes experienced by the system.

In this paper, we deal with the problem of analyzing stochastic multistructural systems with distributed structure changes. Two approaches to solving this problem are considered. These are a statistical simulation method [3], based on modeling of a control system, and a spectral method [4],

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underpinned by the idea of transiting to a deterministic problem (the problem of solving generalized Fokker–Planck–Kolmogorov equations) with a subsequent parametrization of the probability density of the system state vector. Both methods make it possible to estimate any probability characteristics of output processes, probability density in particular.

In analyzing systems with distributed structure changes, the statistical simulation method and the spectral method offer some advantages as compared to some other approximate methods such as, for instance, functional approximation methods (the orthogonal decomposition method and the polygaussian approximation method); see [1, 2]. Using these methods we can pass from the generalized Fokker–Planck–Kolmogorov equations to a system of ordinary differential equations in a rather large dimension for coefficients of a functional series that approximates a probability density or characteristic function. Other approaches, namely, the Gaussian and two-moment parametric approximation methods, call for defining the structure of an a priori unknown probability density.

Both methods, with slight modifications, apply with inexact restorations of realizations [1, 4]. It should be noted that the statistical simulation method is applicable to systems not only with distributed structure changes but also with lumped ones, that is, in the case where a change-over between structures occurs once the state vector has reached a given hypersurface in a space $\mathbb{R}^n$ [1]. The method can also be used with time-sharing systems [5, 6].

In [5], the statistical simulation method and the method of double-stage parametric approximation of probability densities were compared by solving system analysis problems with lumped structure changes. An analysis of numerical calculations made it possible to conclude that both methods give estimates that are in good agreement for integral characteristics of the solution, such as trapping probability, probability density of trapping time, and normalized absorption flux. In calculating the state vector probability density as well as probability moments, better advantage is taken of the statistical simulation method.

A spectral method is more universal as compared to other methods based on orthogonal factorizations. Indeed, relations for solving an analysis problem are algebraic equations which are invariant with respect to choices of basis systems and their properties. Not only are various basis systems used as a method for controlling correctness and accuracy of calculations [7], but they also allow us to derive a solution to an analysis problem with different properties determined by the properties of the functions forming the basis systems.

2. STATEMENT OF AN ANALYSIS PROBLEM

Consider a process $[\boldsymbol{y}(t), s(t)]^{\mathrm{T}}$, where $s(t)$ is a discrete random process with a finite set $\{1, 2, \ldots, S\}$ of states, S is the number of structures in the system, and $\boldsymbol{y}(t)$ is an n-dimensional continuous random process described, under the condition $s(t) = l$, by the following stochastic differential equation (SDE) in the Stratonovich form [1]:

$$d\boldsymbol{y}(t) = a^{(l)}(t, \boldsymbol{y}(t))dt + \sigma^{(l)}(t, \boldsymbol{y}(t))d\mathbf{w}(t), \quad \boldsymbol{y}(t_0) = \boldsymbol{y}_0, \tag{1}$$

or, in the equivalent Ito form,

$$d\boldsymbol{y}(t) = f^{(l)}(t, \boldsymbol{y}(t))dt + \sigma^{(l)}(t, \boldsymbol{y}(t))d\mathbf{w}(t), \quad \boldsymbol{y}(t_0) = \boldsymbol{y}_0. \tag{2}$$

Here $t \in [t_0, T]$; $\boldsymbol{w}(t)$ is an m-dimensional standard Wiener process independent of $\boldsymbol{y}_0$; $a^{(l)}(t, \boldsymbol{y})$, $f^{(l)}(t, \boldsymbol{y})$: $[t_0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ are vector functions of dimension n which are related via

$$f_i^{(l)}(t, \boldsymbol{y}) = a_i^{(l)}(t, \boldsymbol{y}) + \frac{1}{2} \sum_{j_1=1}^{n} \sum_{j_2=1}^{m} \frac{\partial \sigma_{ij_2}^{(l)}(t, \boldsymbol{y})}{\partial y_{j_1}} \sigma_{j_1 j_2}^{(l)}(t, \boldsymbol{y}), \quad i = 1, \ldots, n;$$

$\sigma^{(l)}(t, \boldsymbol{y})$: $[t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n \times m}$ is an $n \times m$ matrix function and l is a structure number, with $l = 1, 2, \ldots, S$ (see [8]).

It should be noted that if $\sigma^{(l)}(t, \boldsymbol{y})$ does not depend on $\boldsymbol{y}$, then the functions $f^{(l)}(t, \boldsymbol{y})$ and $a^{(l)}(t, \boldsymbol{y})$ coincide. In this case, we will not speak of the form in which the equation appears in describing the process $\boldsymbol{y}(t)$ since it is insignificant.